UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 1ST FEBRUARY 2007

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
   Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15.
   When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
   sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
   even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
   rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
   and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

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1. At midnight on 15 December 2005, the moon reached its highest point in the sky, an event which occurs every 18.6 years. In which year will it next occur?

   A 2007   B 2008   C 2023   D 2024   E 2191

2. The information display on a train shows letters by illuminating dots in a rectangular 5 × 8 array. In the letter 't' shown, what fraction of the dots in the array is illuminated?
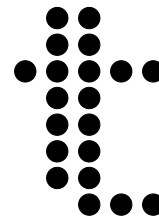
   A $\dfrac{9}{20}$ B $\dfrac{19}{40}$ C $\dfrac{1}{2}$ D $\dfrac{21}{40}$ E $\dfrac{11}{20}$

3. In how many ways can a square be cut in half using a single straight line cut?

   A 1   B 2   C 4   D 8   E Infinitely many

4. Between them, Ginger and Victoria eat two thirds of a cake. If Ginger eats one quarter of the cake, what fraction of the cake does Victoria eat?

   A $\dfrac{1}{2}$ B $\dfrac{2}{5}$ C $\dfrac{3}{8}$ D $\dfrac{4}{9}$ E $\dfrac{5}{12}$

5. What is the value of $(12340 + 12·34) ÷ 1234$?

   A 100.01   B 100.1   C 10.001   D 10.01   E 10.1

6. The sum of 9 consecutive positive whole numbers is 2007. What is the difference between the largest and smallest of these numbers?

   A 8   B 9   C 10   D 18   E 223

7. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are all multiplied together, how many zeros are at the end of the answer?

   A 1   B 2   C 3   D 4   E 10

8. The mean of three numbers x, y and z is x. What is the mean of y and z ?
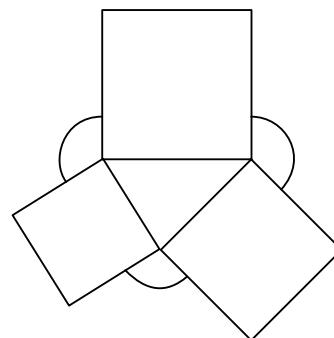
   A $\frac{1}{2}x$ B x C $2x$ D $3x$ E $4x$

9. A male punky fish has 9 stripes and a female punky fish has 8 stripes. I count 86 stripes on the fish in my tank. What is the ratio of male fish to female fish?

   A 2 : 3   B 3 : 2   C 4 : 1   D 4 : 7   E 7 : 4

10. The diagram shows three squares drawn on the sides of a triangle. What is the sum of the three marked angles?

    A 180°   B 270°   C 360°

    D 450°   E It depends on the shape of the triangle

11. The numbers 72, 8, 24, 10, 5, 45, 36, 15 are grouped in pairs so that the product of each pair is the same. Which number is paired with 10?

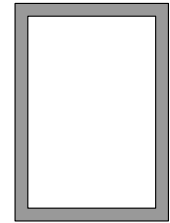A 36   B 45   C 24   D 15   E 72

12. Which of the following could be the graph showing the circumference C of a circle in terms of its diameter d ?

A   B   C   D   E
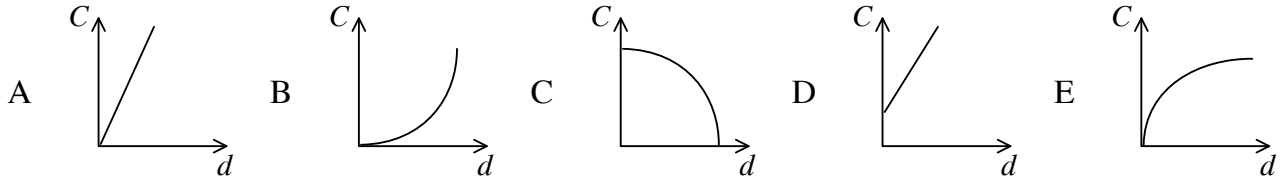
13. A 30 cm × 40 cm page of a book includes a 2 cm margin on each side, as shown.

What percentage of the page is occupied by the margins?

A 14% B 16% C 18% D 20% E 22%

14. If p is a positive integer and q is a negative integer, which of the following is greatest?

A $p - q$ B $q - p$ C $p + q$ D $-p - q$ E More information needed

15. The diagram shows a regular pentagon $PQRST$. The lines QS and RT meet at U. What is the size of angle PUR?

A 108° B 112° C 116° D 126° E 132°
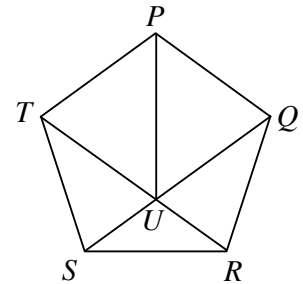
16. A wooden cube with edge length 12 cm is cut into cubes with edge length 1 cm. What is the total length of all the edges of all these centimetre cubes?

A 12 cm   B 12^2 cm   C 12^3 cm   D 12^4 cm   E 12^5 cm

17. Grannie's watch gains 30 minutes every hour, whilst Grandpa's watch loses 30 minutes every hour. At midnight, they both set their watches to the correct time of 12 o'clock. What is the correct time when their two watches next agree?
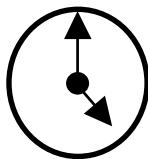
Grannie's watch: Grandpa's watch:
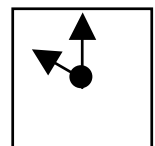
A 6 am   B 9 am   C 12 noon   D 3 pm   E 6 pm

18. One of the digits 1 to 9 is put in each unshaded square so that no digit is repeated and the totals of the entries in the rows and columns are as shown.
What number goes in the starred square?

   A 1   B 3   C 5   D 7   E 9

19. The following sequence continues indefinitely:
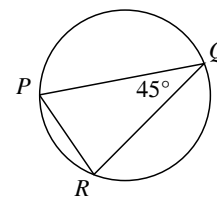
   $27 = 3 \times 3 \times 3,$ $207 = 3 \times 3 \times 23,$ $2\,007 = 3 \times 3 \times 223,$ $20\,007 = 3 \times 3 \times 2\,223, \ldots$

   Which of the following integers is a multiple of 81?

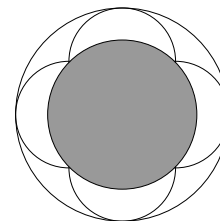   A 200 007   B 20 000 007   C 2 000 000 007 D 200 000 000 007 E 20 000 000 000 007

20. *P*, *Q*, *R* are points on the circumference of a circle of radius 4 cm.
   $\angle PQR = 45°$. What is the length of chord *PR*?

   A 4 cm   B $3\sqrt{3}$ cm   C $4\sqrt{2}$ cm   D 5 cm   E 6 cm

21. The diagram shows two circles and four equal semi-circular arcs.
   The area of the inner shaded circle is 1.
   What is the area of the outer circle?

   A $\sqrt{2}$ B 2   C $1 + \sqrt{2}$ D $\dfrac{\pi}{2}$ E $\dfrac{9}{4}$

22. The diagram shows an ordinary die in which the scores on opposite faces always total 7. It is placed on a horizontal table with the '1' face facing East. The die is moved four times, rotating it each time through 90° about an edge. The faces in contact with the table are first 1, then 2, then 3, then 5. In which direction is the '1' face facing after this sequence of moves?
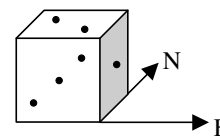
   A West   B East   C North   D South   E Up

23. As *n* takes each positive integer value in turn (that is, *n* = 1, *n* = 2, *n* = 3, and so on) how many different values are obtained for the remainder when n^2 is divided by *n* + 4?

   A 1   B 8   C 9   D 16   E Infinitely many

24. In the diagram on the right, how many squares, of any size, are there whose entries add up to an even total?

   A 12   B 20   C 32   D 36   E 45

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

25. The diagram shows a semi-circle and an isosceles triangle which have equal areas. What is the value of $\tan x°$?

   A 1   B $\dfrac{\sqrt{3}}{2}$ C $\dfrac{\pi}{\sqrt{3}}$ D $\dfrac{2}{\pi}$ E $\dfrac{\pi}{2}$